Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans) B

let time taken for service transmission= T

T is normally distributed with *μ* = 45 minutes and standard deviation *σ* = 8 minutes.

Time delay= 10 minutes

Time available to finish the work= 60-10=50 minutes.

Therefore from the equation Z=(T-µ)/ *σ*

P(T≤50)=p(Z≤(50-45)/8)=p(Z≤0.625)= 0.7324(using z table)

Therefore p(T>50)=1-p(≤50)= 1-0.7324= 0.2676

(Or)

Using R-function : [1-pnorm(50,45,8)]

Or python using below:

|  |
| --- |
| > 1-stats.norm.cdf(50, loc =45, scale = 8 )  0.2659855 |
|  |
| |  | | --- | |  | |

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans) False.

• 68% of the data falls within one standard deviation of the mean (µ+*σ)*.

Here µ=38, *σ* =6

Then, µ+*σ= 38+*6=44

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans) True

Z=(X-µ)/ *σ*

P(X≤30)=p(Z≤(30-38)/6)=p(Z≤-1.33)= 0.0918(using z table)

Expected count=0.0918\*400= 36.72

1. If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid*normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans) 2 is simply a larger scale version of the random variable *X1.* If is normally distributed then 2X1 is also normally distributed.

*X*1 and *X*2 are normal distributed, the associated sums and random samples are exactly (and not just approximately) normal, with the appropriate parameters.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans) D

Here we need range of 99% data which lies between 3rd standard deviation of the mean.

Here µ=100, *σ* =20

From empirical rule, µ±3*σ= 100±3\*20=>(100-60, 100+60)=>(40,160).*

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans) let, X is the sum of two random variables having normal distribution.

E[X]= E[45\*(profit 1+profit 2)]= 45\*(5+7)=540 million rupees

SD[X]= SD[profit 1 +profit 2]=> 45\*()

= 45\*= 225 million rupees.

**Therefore, X~ N(540,)**

1. From the empirical rule, Approximately 95% of the data falls within two standard deviation of the mean.

μ ± 2σ = 540±2\*225=> (540-450, 540+450)=> **(90,990)**

**B)**



From the above normal distribution we can say that to find 5th percentile from the left side we can use the formula,

μ - 1.5σ => 540-(1.5\*225) =>202.5 million rupees.

c) this question concerns the original profit distributions.

For division1= Z score for a profit of zero: Z=(X-µ)/ *σ =>*  (0-5)/3 => -1.66=0.0485

(or)

> stats.norm.cdf(0, loc =5, scale = 3 )

0.04779035

For division2= Z score for a profit of zero: Z=(X-µ)/ *σ*  =(0-7)/4 => -1.75= .0401

> stats.norm.cdf(0, loc =7, scale = 4 )

0.04005916

Division2 has a higher probability of making a loss.